



A system dynamics approach to modelling dairy cattle production structure and herd dynamics

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Abstract

Understanding herd demographic structure is fundamental for evaluating productivity, disease dynamics, and management interventions in dairy systems. This study developed a dynamic production subsystem to represent the progression of female cattle through key physiological and production stages using a system dynamics modelling approach. The subsystem captures transitions from calves to heifers, pregnant heifers, and successive lactations (L1–L4+), incorporating biologically realistic delays for maturation, gestation, calving, and parity progression. The model was formulated using ordinary differential equations and simulated over a 10-year period to assess internal consistency, numerical stability and long term behaviour. Simulation results demonstrated coherent population dynamics with orderly progression through life stages and no biologically unrealistic accumulation in higher parities. Calf and heifer stocks showed expected transient behaviour, while lactation stocks exhibited staggered peaks corresponding to cumulative biological delays and attrition due to culling and mortality. The absence of oscillations and negative stock values indicates numerical stability and preservation of mass balance. The resulting parity-wise herd structure closely reflects patterns observed under stable dairy management conditions. The production subsystem provides a robust demographic foundation for integration with epidemiological and economic models, particularly for production diseases such as mastitis where risk and losses vary by lactation stage. By explicitly representing physiological delays and population turnover, the model enables evaluation of long-term management strategies and their delayed effects on herd structure. This subsystem can be readily extended to support integrated analyses of productivity, disease control, and sustainability in dairy production systems.

Keywords: System dynamics, dairy cattle, herd demography, production subsystem

Dairy production systems operate as complex biological and management-driven systems in which herd structure evolves continuously over time. The number of animals in different age and parity classes determines milk

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production potential, reproductive efficiency, disease susceptibility, and economic outcomes (Van et al., 2020). Changes in management practices, disease control strategies, or replacement policies often exert delayed effects on herd performance, making intuitive assessment difficult without formal modelling approaches.

Traditional static or snapshot-based analyses may fail to capture the cumulative and time-dependent nature of demographic processes in dairy herds. In contrast, dynamic models allow explicit representation of flows between life stages and provide insights into how short-term decisions influence long-term outcomes (Tuljapurkar et al., 2009). In particular, system dynamics modelling has been increasingly applied to agricultural and veterinary systems due to its ability to represent feedback loops, delays, and stock–flow relationships. For diseases such as mastitis, where incidence and economic impact vary by lactation stage and parity, an accurate representation of herd demography is essential. Without a biologically realistic production subsystem, integrated disease or economic models risk generating misleading results driven by structural artefacts rather than true system behaviour implying structural validity and realistic subsystem design are essential to avoid artefacts and false policy conclusions in integrated models (Rahmandad et al., 2012). Despite this, many existing models simplify herd structure excessively or ignore physiological delays.

The present study addresses this gap by developing a dynamic production subsystem representing the female dairy cattle population. The objective was to construct a biologically coherent and numerically stable demographic model that can serve as a foundational component for integrated analyses of productivity, disease

dynamics, and management strategies.

Materials and methods

Model structure

The production subsystem was conceptualised as a closed population model representing the major life stages of female dairy cattle. The model included state variables (stock) representing calves, heifers, pregnant heifers, and lactating cows stratified by parity into first (L1), second (L2), third (L3), and fourth and higher lactations (L4+). This structure reflects key physiological transitions relevant to production performance and disease risk (Fig.1).

Flows between stocks were governed by transition processes including maturation, conception, parity progression, mortality, and exit from the system through culling. Explicit delays were incorporated to ensure that animals remained in each physiological state for biologically realistic durations. The aggregation of higher parities into a single stock was adopted to balance biological realism with model simplicity and transparency.

Mathematical formulation

The production subsystem was formulated as a system of ordinary differential equations, with each stock representing the integral of its inflows minus outflows over time. Transition rates were parameterised to reflect average biological processes under stable dairy management conditions. Fixed delays were used to represent gestation and age at first calving, preventing premature movement of animals between states.

Production Sub-Model Representing Herd Demographic and Lactation Dynamics

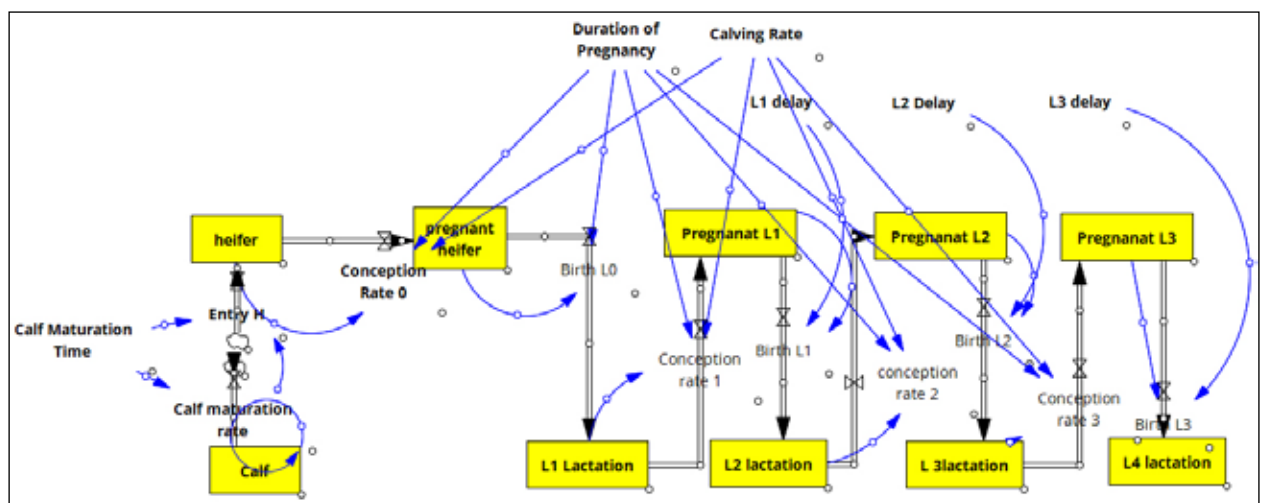


Fig. 1 : The production sub-model depicts the demographic progression of animals through calf, heifer, pregnancy, and successive lactation stages (L1–L4). Transitions between stocks are governed by biologically defined parameters such as calf maturation time, conception rates, duration of pregnancy, calving rate, and lactation delays. The model captures the continuous flow of animals across reproductive and productive stages, forming the structural basis for evaluating milk production and disease dynamics within the herd.

Mortality and culling were represented as continuous outflows from each stock, contributing to an exit stock that captured removal from the production system. This formulation ensured conservation of mass (population balance) and avoided unrealistic accumulation or depletion of population stocks.

Mathematical formulation

Calves

$$\frac{dCalf(t)}{dt} = -\text{Calf Maturation Rate}(t)$$

with initial condition:

$$Calf(0) = 100$$

The calf maturation rate is defined as:

$$\text{Calf Maturation Rate}(t) = \min\left(\frac{Calf(t)}{T_m}, Calf(t)\right)$$

where $T_m = 547$ days is the calf maturation time.

Heifers

Entry into the heifer stock occurs after a fixed maturation delay:

$$Entry_H(t) = \text{DELAY}_{fixed}(\text{Calf Maturation Rate}(t), T_m)$$

The heifer stock evolves as:

$$\frac{dHeifer(t)}{dt} = Entry_H(t) - CR_0(t)$$

where the heifer conception rate is:

$$CR_0(t) = \frac{Heifer(t) \times \text{Calving Rate}}{T_p}$$

$$CR_0(t) = \frac{Heifer(t) \times \text{Calving Rate}}{T_p} \quad \text{and}$$

$$\text{Calving Rate} = 0.815 \quad \text{Calving Rate} = 0.815, \\ T_p = 280T_p = 280 \text{ days (duration of pregnancy).}$$

Pregnancy and calving processes

Pregnant heifers (L0)

$$\frac{d\text{Preg Heifer}(t)}{dt} = CR_0(t) - \text{Birth}_{L0}(t)$$

with

birth flow:

$$\text{Birth}_{L0}(t) = \frac{\text{PregHeifer}(t)}{T_H}$$

where $T_H = 900T_H = 900$ days is the heifer delay.

Lactation and parity progression

First lactation (L1)

$$\frac{dL1(t)}{dt} = \text{Birth}_{L0}(t) - CR_1(t) - \text{CMR}_1(t) - \text{SCMR}_1(t)$$

The conception rate during L1 is:

$$CR_1(t) = \frac{0.476 \times L1(t) \times \text{CalvingRate}}{T_p}$$

Pregnant L1

$$\frac{d\text{Preg L1}(t)}{dt} = CR_1(t) + \text{Entry}_{L1R}(t) - \text{Birth}_{L1}(t) - \text{CMR}_{1P}(t) - \text{SCMR}_{1d}(t)$$

with:

$$\text{Birth}_{L1}(t) = \frac{\text{Preg L1}(t)}{T_{L1}}$$

where $T_{L1} = 1200T_{L1} = 1200$ days.

Second lactation (L2)

$$\frac{dL2(t)}{dt} = \text{Birth}_{L1}(t) - CR_2(t) - \text{CMR}_2(t) - \text{SCMR}_2(t)$$

$$CR_2(t) = \frac{0.482 \times L2(t) \times \text{CalvingRate}}{T_p}$$

Pregnant L2

$$\frac{d\text{Preg L2}(t)}{dt} = CR_2(t) + \text{Entry}_{L2R}(t) - \text{Birth}_{L2}(t) - \text{CMR}_{2P}(t) - \text{SCMR}_{2P}(t)$$

$$\text{Birth}_{L2}(t) = \frac{\text{Preg L2}(t)}{T_{L2}}$$

where $T_{L2} = 1500T_{L2} = 1500$ days.

Third lactation (L3)

$$\frac{dL3(t)}{dt} = \text{Birth}_{L2}(t) - CR_3(t) - \text{CMR}_3(t) -$$

$$CR_3(t) = \frac{0.493 \times L3(t) \times \text{Calving Rate}}{T_p}$$

Pregnant L3

$$\frac{d\text{Preg L3}(t)}{dt} = CR_3(t) + \text{Entry}_{L3R}(t) - \text{Birth}_{L3}(t) - \text{CMR}_{3P}(t) - \text{SCMR}_{3P}(t)$$

$$\text{Birth}_{L3}(t) = \frac{\text{Preg L3}(t)}{T_{L3}}$$

where $T_{L3} = 1800T_{L3} = 1800$ days.

Fourth and higher lactations (L4+)

$$\frac{dL4(t)}{dt} = \text{Birth}_{L3}(t) - \text{CMR}_4(t) - \text{SCMR}_4(t)$$

Delay-based re-entry after recovery

Recovered animals re-enter lactation after parity-specific delays:

$$Entry_{L1R}(t) = DELAY_{fixed}(L1_R(t), D_{L1})$$

$$Entry_{L2R}(t) = DELAY_{fixed}(L2_R(t), D_{L2})$$

$$Entry_{L3R}(t) = DELAY_{fixed}(L3_R(t), D_{L3})$$

with:

$$D_{L1} = 540 + 365, D_{L2} = 540 + 2 \times 365, D_{L3} = 540 + 3 \times 365$$

Simulation settings

The model was implemented using a system dynamics simulation platform and run over a 10-year period to capture long-term demographic behaviour. Initial conditions represented a finite cohort of female calves entering the system. Model outputs were examined graphically and numerically to assess internal consistency, stability, and biological plausibility. The model is simulated over a 10-year horizon:

$$t \in [0, 3650] \text{ days}$$

with a fixed time step: $\Delta t = 1 \text{ day}$

Parameterisation and Assumptions

Model parameterisation for the production subsystem was based on published literature and standard biological assumptions. Calf maturation time was assumed to be 547 days, representing the average duration required for female calves to reach reproductive maturity under typical dairy management conditions. Conception rates were specified for different physiological stages and lactation sequences based on empirical estimates reported by Bhagat et al. (2019). The conception rate for heifers was set at 49.22 per cent, reflecting reproductive

performance at first breeding. Among lactating cows, conception rates were defined as 47.63 per cent in first lactation, 48.20 per cent in second lactation, and 47.71 per cent in third lactation. Higher parity cows showed comparable reproductive efficiency, with conception rates of 49.30 per cent in fourth lactation and 48.12 per cent in fifth and subsequent lactations. These parity-specific values ensured biologically realistic transitions between lactation and pregnancy states and allowed the model to capture gradual changes in reproductive performance across successive parities (Martin et al., 2019).

Results and discussion

The overall structure of the production subsystem (Fig. 1) ensured sequential and biologically appropriate progression of animals through calves, heifers, pregnancy, and successive lactations. Simulation outputs over the 10-year period (Fig. 2 and 3) results demonstrated smooth and continuous transitions consistent with expected biological dynamics.

The calf population declined steadily as animals matured into heifers, approaching zero towards the later stages of the simulation due to the finite initial cohort and absence of external recruitment. Such dynamics are characteristic of first-order aging chains, in which outflows are proportional to the existing stock, leading to exponential decay when no additional inflows are present (Serman, 2000; Ford, 2010). Heifer stocks increased initially and reached a peak before declining as animals transitioned into pregnancy. This pattern is consistent with expected demographic behaviour under closed herd conditions.

Pregnant heifers exhibited a delayed peak relative to non-pregnant heifers, confirming appropriate implementation of gestation delays. Entry into first lactation occurred within biologically realistic timeframes, corresponding to recommended ages at first calving in dairy cattle (Tozer & Heinrichs, 2001). These findings

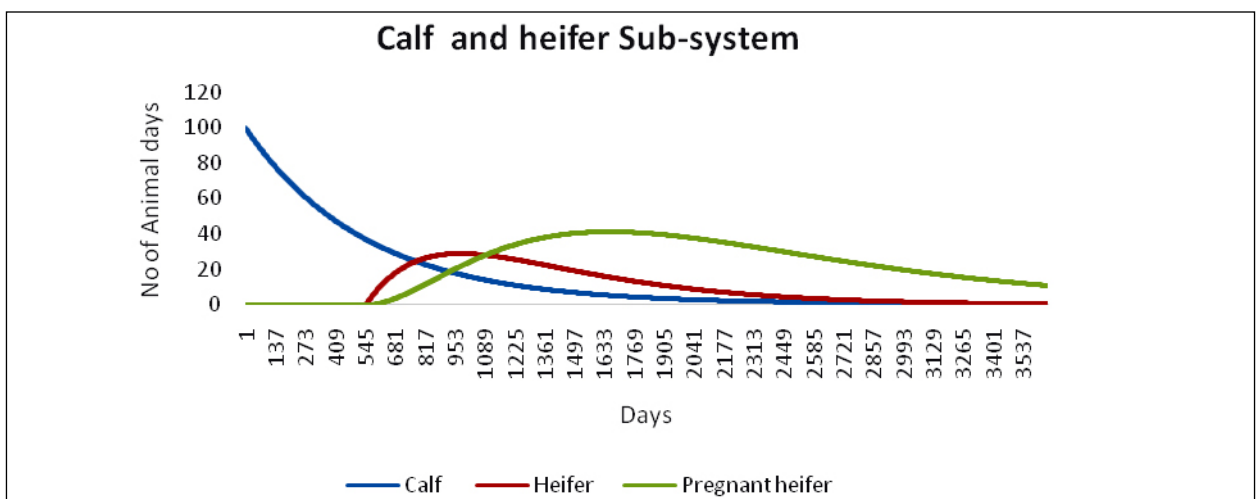


Fig. 2 : Calf and heifer Sub-system

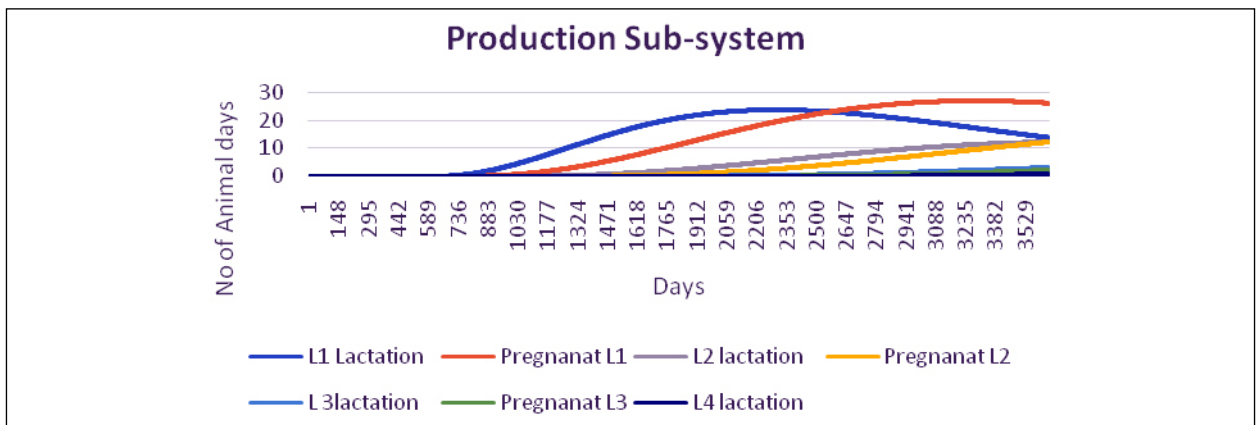


Fig. 3 : Production Sub-system

suggest that maturation and reproductive processes were represented with reasonable biological accuracy.

Parity-wise lactation dynamics

Lactating cow stocks displayed staggered peaks across successive parities, with each higher lactation peaking later and at a lower magnitude than the preceding one. This behaviour reflects cumulative biological delays associated with calving intervals, as well as progressive attrition due to mortality and culling. The aggregation of fourth and higher lactations into a single L4+ stock prevented excessive fragmentation while retaining a realistic representation of mature cows. The stabilization of the L4-and-above compartment indicates that the model achieved a sustainable parity structure without unrealistic accumulation or depletion of older cows. This was particularly important given the significant contribution of multiparous cows to total milk yield and farm profitability (De Vries, 2006).

Relevance for production disease and management modelling

From a practical standpoint, the ability of the model to generate realistic, time-varying parity distributions is particularly important for modelling production diseases such as mastitis. Parity-based compartmentalization is critical for accurately representing herd productivity and long-term demographic stability (De Vries, 2006). Disease risk, milk yield loss, and treatment decisions differ substantially between early and late lactation cows, as well as across parities. Early and peak lactation periods are associated with greater metabolic stress and higher milk losses during disease perturbations, and parity (first vs. later lactations) influences both susceptibility and recovery patterns (Adriaens et al., 2021).

The production subsystem provides the necessary demographic resolution to support such analyses. Moreover, the explicit representation of delays enables evaluation of long-term impacts of interventions, such as improved heifer management or altered culling

strategies, which may take several years to manifest at the herd level (Eurich et al., 2005).

The model assumes homogeneity within each stock (compartment), which may not fully capture individual-level variability. While this limits representation of short-term variability, the approach is appropriate for analysing long-term structural behaviour and policy-relevant outcomes. The closed population assumption simplifies interpretation of internal dynamics and enhances transparency. Although the model captures structural dynamics effectively, inclusion of stochastic variability and external replacement flows in future work would enhance its applicability to real-world dairy systems.

Conclusion

The dynamic production subsystem developed in this study provides a biologically coherent and numerically stable representation under the assumptions considered. By explicitly modelling physiological delays and parity progression, the subsystem generates realistic herd structures suitable for long-term simulation. This production module offers a robust foundation for integrated epidemiological and economic models aimed at improving productivity, disease control, and sustainability in dairy production systems.

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